A Classification of Toroidal Orientifold Models

P. Anastasopoulos[1,α,β], A. B. Hammou[2,α,γ]

α Department of Physics, University of Crete,
71003 Heraklion, GREECE.

β Laboratoire de Physique Théorique Ecole Polytechnique,
91128 Palaiseau, FRANCE.

γ Departement de Physique, Université des Sciences et Technologie d'Oran,
BP 1505, Oran, El M'Naouer, ALGERIE

Abstract

We develop the general tools for model building with orientifolds, including SS supersymmetry breaking. In this paper, we work out the general formulae of the tadpole conditions for a class of non supersymmetric orientifold models of type IIB string theory compactified on T^6, based on the general properties of the orientifold group elements. By solving the tadpoles we obtain the general anomaly free massless spectrum.

[1]panasta@physics.uoc.gr
[2]amine@physics.uoc.gr

1 Introduction

Orientifolds are a generalization of orbifolds [1, 2, 3], where the orbifold symmetry includes orientation reversal on the worldsheet. The orientifold group contains elements of two kinds: internal symmetries of the worldsheet theory forming a group G_1 and elements of the form $\Omega \cdot g$, where Ω is the worldsheet parity transformation and g is some symmetry element which belongs to a group G_2. Closure implies that $\Omega \cdot g \cdot \Omega \cdot g' \in G_1$ for $g, g' \in G_2$. The full orientifold group is $G_1 + \Omega G_2$. The one loop amplitude implementing the Ω projection is interpreted as Klein Bottle amplitude and has in general ultraviolet divergences (tadpoles). These ultraviolet divergences are interpreted as sources in space time, that couple to the massless type IIB fields, the metric, the dilaton and the R-R forms. They are localized in sub-manifolds of space-time, known as *orientifold planes*, denoted by O_p. These are non dynamical objects characterized by their charges and tensions. Consistency and stability of the theory are assured if D-branes are introduced in a way that guarantees the cancellation of these tadpoles. R-R tadpole cancellation is equivalent to the vanishing of gauge charge in a compact space, whereas NS-tadpole cancellation is equivalent to the vanishing of forces in the D-brane/O-plane vacuum configuration [4].

In this work, primarily we study the tadpole conditions for a class of orientifold groups of the type $G + \Omega\, G$, where G is an orbifold group which contains only geometrical rotation elements that preserve some supersymmetry.

All the information of the type and the positions of the orientifold planes are encoded in the Klein bottle amplitudes. We realize easily that only some specific type of twisted closed strings couple to the O-planes. In this work, we classify the contribution of the O-planes to the tadpoles by studying the general properties of the elements of the orbifold group G. This classification shows that only elements $\alpha \in G$ such that there exist an element $\beta \in G$ with $\alpha = \beta^2$ give non trivial contribution to the tadpoles.

To cancel the aforementioned Klein Bottle UV divergences, we need to add proper D-branes to the closed string sector (open string sector). Moreover, the type of D-branes needed in each of the models together with their contributions to the massless tadpoles can be similarly classified.

Therefore, we provide general tadpole conditions by studying the general properties of the elements in G. Our general results for supersymmetric orientifolds agree with the tadpole conditions of the models already studied in the literature [6, 12, 13].

Following the same spirit, we enlarge our study to orientifold models with spontaneous supersymmetry breaking. For these models, G contains in addition to rotation elements, also freely acting ones that break supersymmetry a la Scherk-Schwarz. For simplicity, the freely acting part we consider is a particular Scherk-Schwarz deformation by a momentum

shift of order two, accompanied by $(-1)^F$, where F is the spacetime fermion number. We focus on abelian orientifold groups G where freely and non-freely acting elements commute. Therefore, since the freely acting elements are Z_2 translations, we restrict our selves for simplicity only to Z_2 rotation factors. Studying the Klein Bottle amplitude in this case, we realize that in addition to the usual O-planes, we obtain anti-$\bar{O}$-planes when Z_2 rotation elements act longitudinal to the Scherk-Schwarz deformation element. To cancel all tadpoles we need to add D-branes as well as $\bar{D}$-antibranes. Finally, we work out the tadpole conditions based on the general properties of the elements of G. A generalization to other Scherk-Schwarz deformations, as winding shift or of the type considered in [8], is not too difficult to be achieved.

Next step in our study is to solve the tadpole conditions. The action of the orientifold group on the Chan-Paton factors is made by the γ_α matrices. In general, these matrices obey to the tadpole conditions and to $\gamma_\alpha^N = \pm 1$ where N is the smallest integer for which this equation holds. In general, we can go to a basis where γ_α is diagonal with entries the Nth roots of unity ± 1. The number of times each entry appears depends on the tadpole conditions. However, we can give the general spectra of the models based on the general properties of the orientifold group. Since the Scherk-Schwarz commutes with the rotation elements, we can diagonalize the γ_h matrix and study the way it breaks super-symmetry. Therefore, we provide the general effect of this element to the supersymmetric representations.

This paper is organized as follows. In section two we present some generalities about the orientifold group we are using and discuss the general form of the elements according to supersymmetry. In section three we consider in some details the calculations of the tadpole conditions for the supersymmetric orientifold models. In section four we discuss the breaking of supersymmetry with a Scherk-Schwarz deformation. In section five we give some applications and compare our results with some of the models existing in the literature. In section six we discuss the general solutions of the tadpole conditions and provide the general spectra for supersymmetric and non-supersymmetric orientifold models. The details of the calculations are presented in the appendices.

2   The Setup

Consider the projection of type IIB string theory on $R^4 \times T^6$ by the orientifold group $G + \Omega\, G$. The orbifold group G contains rotation and translation elements and acts on the 6-dimensional torus T^6, which we factorize as $T_1^2 \times T_2^2 \times T_3^2$. The orbifold action on the complex coordinates z^i of the three tori with $i = 1, 2, 3$ is given by

$$\alpha = e^{2\pi i \sum_{i=1}^{3} (v_\alpha^i J^i + R_i\, \delta_\alpha^i\, P^i)} \, , \tag{1}$$

with J^i and P^i the generators of rotations and diagonal translation in each of the internal two torus T_i^2 with radius R_i. Let us recall some facts about supersymmetries in four-dimensional orbifold compactifications [8]. The basic Majorana-Weyl supercharge Q in $D = 10$ fills the **16** of $SO(9,1)$. This decomposes in $D = 4$ into four Majorana supercharges $Q_n = Q_{nL} + Q_{nR}$, transforming each as a $\mathbf{2} \oplus \bar{\mathbf{2}}$ under $SO(3,1)$ and together as a **4** of the maximal $SO(6)$ R-symmetry group. For each $n = 1, 2, 3, 4$, Q_{nL} and Q_{nR} have $SO(6)$ weights w_n and $-w_n$ respectively, where:

$$w_1 = \left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\right), \quad w_2 = \left(\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}\right), \quad w_3 = \left(-\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}\right), \quad w_4 = \left(-\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{2}\right). \tag{2}$$

A generic orbifold element α acts as the combination of a rotation of angle $2\pi v_i$ and some unspecified shift, in each of the 3 internal T_i^2. Under this action, the four possible supercharges transform as:

$$
\begin{aligned}
Q_{nL} &\to e^{2\pi i v \cdot w_n} Q_{nL}, \\
Q_{nR} &\to e^{-2\pi i v \cdot w_n} Q_{nR}.
\end{aligned}
\tag{3}
$$

Therefore, the supercharge Q_n is left invariant by α if $v \cdot w_n$ is an integer, independently of the shift.

For the translation we will consider in this paper only shifts of order two together with $(-1)^F$, where F is the space-time fermion number, this element will be denoted through all the paper by h. This is a freely acting orbifold and is a particular Scherk-Schwarz (SS) deformation, which breaks supersymmetry spontaneously by giving a mass to the fermions. In general, the Scherk-Schwarz deformation can be implemented by an order n shift together with a rotation of the same order that breaks supersymmetry [8, 9] i.e. an element v such that $\sum_i v_i \neq 0$.

In the direction where a shift acts the only allowed rotations are those that commute with it. Therefore, in a direction where an order two shift acts, we will consider at most a rotation by a Z_2 element, denoted by R_j (R_j : $z^i \to e^{2\pi i g_i(1-\delta_{ij})} z^i$ where $g_i = \pm\frac{1}{2}$. Notice that R_j does not act on the torus T_j^2.

Taking into account all these considerations we realize that in the supersymmetric case, the most general rotation elements α are such that $v_\alpha = (v_\alpha^1, v_\alpha^2, v_\alpha^3)$ with say $v_\alpha^3 = 0$ or $v_\alpha^3 \neq 0$. On the other hand, in the non-supersymmetric cases where we break supersymmetry by a SS deformation element that acts on the last torus, the most general rotation elements are of the form $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$ or $v_\alpha = (v_\alpha^1, v_\alpha^2, \frac{1}{2})$. The former can be written as $v_\beta + g_{i=1,2}$, where $v_\beta^3 = 0$ a rotation in the $T_1^2 \times T_2^2$ torus and $g_{i=1,2}$ a Z_2 rotation element. Therefore, without loss of generality we can take α such that $v_\alpha^3 = 0$.

We would like to study the contribution to the tadpole conditions of a generic element $\alpha \in G$. The divergences can be determined from the vacuum amplitudes on the Klein

Bottle ($\mathcal{K}$), Annulus ($\mathcal{A}$) and Möbius strip ($\mathcal{M}$). We will consider two different cases: supersymmetric orientifolds (without SS $h \notin G$) and non-supersymmetric orientifolds (with SS $h \in G$).

3   Supersymmetric Orientifolds

Klein Bottle

Let us consider an element α of G. We can work out the contribution of this element to the Klein Bottle amplitude by using the trace formula:

$$\mathcal{K}_\alpha = Tr_{U+T} \left[\Omega\alpha \ \ q^{L_0} \bar{q}^{\bar{L}_0} \right] \ , \tag{4}$$

where the subscripts U and T refers to the untwisted and twisted closed string states of type IIB orbifold model considered. Due to the presence of Ω the only states contributing to $\mathcal{K}_\alpha$ are the untwisted states and the Z_2 twisted ones. We remind that by Z_2 we mean an order two rotation elements i.e. $R^2 = 1$. This element acts always on two tori since we do not want to break completely supersymmetry. The twisted states exist only if the orbifold group G contains Z_2 factors.

Since the Klein Bottle is equivalent to a cylinder with two crosscaps (figure.1), then the contribution of a group element α corresponds to a propagation of a close string state projected by $(\Omega\alpha)^2 = \alpha^2$ for group elements that commute with Ω [3]. If the orbifold group G contains a Z_2 factor denoted by R, it produces an extra contribution since $(\Omega R\alpha)^2 = \alpha^2$. To extract the massless tadpole contribution we perform a modular transformation $l = 1/4t$ where t is the loop modulus and l the cylinder length [5, 6] and then take the limit $l \to \infty$.

- The contribution to the tadpoles from the Klein Bottle amplitudes of a group element α such that $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$ is:

  - When the orbifold group G does not include an R factor, the only contribution will come from the untwisted sector states:

  $$(1_{NS} - 1_R)\mathcal{V}_3 \frac{1}{\prod_l 2 \ \sin 2\pi v_\alpha^l} \left(\prod_l 2 \ \cos \pi v_\alpha^l \right)^2 \ , \tag{5}$$

  where the factor $\prod_l 2 \ \sin 2\pi v_\alpha^l$ is related to the action of α on Neumann directions [5, 6] and $\mathcal{V}_3$ is the regularized volume of the third torus T_3^2.

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[3]It is because we consider the orientifold group to be $G + \Omega G$ which implies that Ω commutes with all elements of G
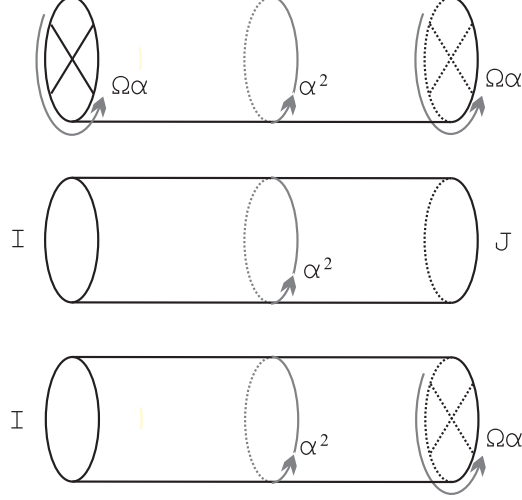
Figure 1: Klein-bottle, Annulus and Möbius strip. The one-loop amplitudes become tree-level in the transverse picture where an α^2-twisted closed string propagates between crosscaps and boundaries.

- On the other hand, if the group G contains an R factor, we have extra contributions from the R-twisted states. The full contribution for the different cases is:

  i. If there is only one Z_2 element that act parallel to v_α (only $R_3 \in G$):

  $$(1_{NS} - 1_R)\frac{1}{\prod_l 2\,\sin 2\pi v_\alpha^l}\mathcal{V}_3\Big(\prod_l 2\,\cos \pi v_\alpha^l + \prod_l 2\,\sin \pi v_\alpha^l\Big)^2. \qquad (6)$$

  ii. If there is only one $R_i \in G$ for a given $i = 1$ or 2 (which Z_2 factor acts also on the third torus T_3^2):

  $$(1_{NS} - 1_R)\frac{1}{\prod_l 2\,\sin 2\pi v_\alpha^l}\Big(\sqrt{\mathcal{V}_3}\prod_l 2\cos \pi v_\alpha^l - \frac{1}{\sqrt{\mathcal{V}_3}}2\cos \pi v_\alpha^i 2\sin \pi v_\alpha^j\Big)^2. \quad (7)$$

  iii. If there are all three possible Z_2 factors $R_l \in G$ with $l = 1, 2, 3$:

  $$(1_{NS} - 1_R)\frac{1}{\prod_l 2\sin 2\pi v_\alpha^l}\Bigg\{\sqrt{\mathcal{V}_3}\Big(\prod_l 2\cos \pi v_\alpha^l + \prod_l 2\sin \pi v_\alpha^l\Big)$$
  $$- \frac{1}{\sqrt{\mathcal{V}_3}}\sum_{i \neq j = 1,2} \epsilon_{ij}\,2\cos \pi v_\alpha^i 2\sin \pi v_\alpha^j\Bigg\}^2. \qquad (8)$$

  where $\epsilon_{12} = -\epsilon_{21} = 1$.

All the amplitudes above are proportional to $(1_{NS} - 1_R)$ and their multiplicatives appear as perfect squares [2, 7]. We should mention that for this kind of orbifold

action all the amplitudes are volume dependant $(\mathcal{V}_3)$. They are of the general form:

$$(1_{NS} - 1_R)\left[K_1\sqrt{\mathcal{V}_3} + \frac{K_2}{\sqrt{\mathcal{V}_3}}\right]^2 , \tag{9}$$

where K_1 and K_2 are constants encoding the information about the orbifold projection.

- Consider now an element that acts on all the three tori $v_\alpha = (v_\alpha^1, v_\alpha^2, v_\alpha^3)$ where $v_\alpha^{l=1,2,3} \neq 0$ or $1/2$. The contribution to the tadpoles depends again on the existence of an R factor in G:

  - If G contains no R factors:

  $$(1_{NS} - 1_R)\frac{1}{\prod_l 2 \ \sin 2\pi v_\alpha^l}\left(\prod_l 2 \ \cos \pi v_\alpha^l\right)^2 . \tag{10}$$

  - If G contains R factors, then:

    i. if it contains only one Z_2 factor $R_i \in G$ for a given i:

    $$(1_{NS} - 1_R)\frac{1}{\prod_l 2 \ \sin 2\pi v_\alpha^l}\left(\prod_l 2 \ \cos \pi v_\alpha^l + 2 \ \cos \pi v_\alpha^i \prod_{l\neq i} 2 \ \sin \pi v_\alpha^l\right)^2 . \tag{11}$$

    ii. If G contains all three possible Z_2 factors $R_l \in G$ for $l = 1, 2, 3$:

    $$(1_{NS} - 1_R)\frac{1}{\prod_l 2\sin 2\pi v_\alpha^l}\left(\prod_l 2\cos \pi v_\alpha^l + \sum_i 2\cos \pi v_\alpha^i \prod_{l\neq i} 2\sin \pi v_\alpha^l\right)^2 . \tag{12}$$

All the amplitudes are again perfect squares as they should [1, 2, 7].

In orientifold models, tadpoles arise as divergences at the one-loop level, which are interpreted as inconsistencies in the field equations for the R-R potentials in the theory. These tadpoles can be regarded as the emission of an R-R closed string state from a Dp-brane (disc), a source of (p+1)-form R-R potential and from an orientifold plane ($\mathbf{RP^2}$), which carries R-R charges. The contributions we have obtained above come from Op-planes sources of twisted (p+1)-form R-R potentials. For non trivial twists, the string zero modes vanish, and therefore the Dp-brane is forced to sit at the fixed point. It simply means that the twisted R-R potential can not propagate in spacetime. Depending on the tension and charge, there are four kinds of orientifold planes: O^+ (O^-) with negative (positive) tension and charge and $\bar{O}^+$ ($\bar{O}^-$) with negative (positive) tension and positive (negative) charge [7]. In this paper we will meet three kinds of these orientifold-planes, namely O^-, $\bar{O}^-$ and O^+. The later appear in models with broken supersymmetry by an order two Scherk-Schwarz deformation [7, 8, 10]. The $\bar{O}^+$ orientifold planes appear in orientifold models of type IIB string theory with a discrete NS antisymmetric tensor switched on (discreet torsion) [10].
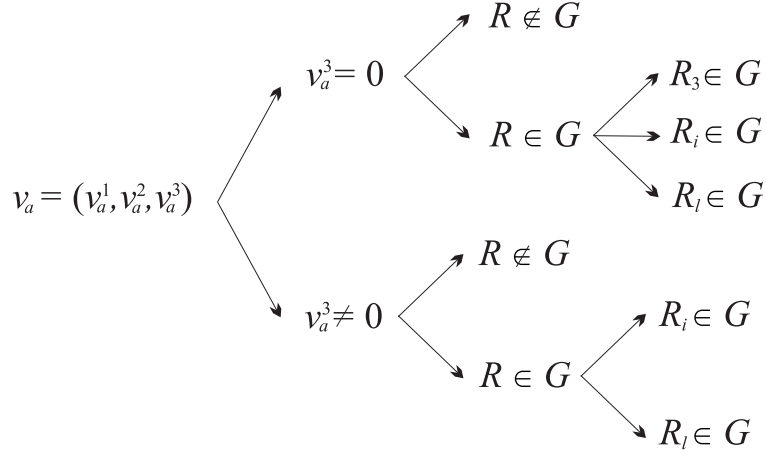
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Figure 2: We classify the tadpole conditions according to the kind of elements belonging to the orbifold group G. $i = 1$ or 2 and $l = 1, 2, 3$.

Annulus

To cancel the aforementioned Klein Bottle UV divergences -tadpoles- we need to add D-branes to the spectrum (open string sector). These are a bunch of D9-branes and D5$_i$-branes extended along the T_i^2 torus and siting on the R_i-fixed points when the group G contains R_i-factors [4]. The Annulus amplitudes can be easily computed for all kinds of D-branes existing in the theory, with the contribution of the group element α given by the trace formula:

$$\mathcal{A}_{IJ,\alpha} = Tr_{IJ} \left[\alpha \, q^{L_0} \right] \ , \tag{13}$$

where now the trace is over all open string states attached between I and J D-branes for $I, J = 9, 5_i$ siting at α-fixed point. To extract the tadpole contributions we need to perform a modular transformation to the transverse channel $l = 1/2t$ and then take the limit $l \to \infty$ [5].

Similar to the Klein-bottle cases, we classify the contributions to the tadpoles by the existence of Z_2 rotation elements in G (figure.2):

- For an element α such that: $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$ we have the following contribution to the tadpoles:

  - if G contains no Z_2-factors, the only contribution to the annulus amplitude is coming from the 99 strings.

$$(1_{NS} - 1_R)\mathcal{V}_3 \frac{1}{\prod_l 2 \, \sin \pi v_\alpha^l} Tr[\gamma_{\alpha,9}]^2 \ . \tag{14}$$

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[4]We should mention that D7 and D3 branes are obtained in orientifold models where Ω acts together with a reflection element I_2 that gives a minus sign to the coordinates of one of the T_i^2 torus.

- if G contains R-factors, then we have also contributions from the corresponding D5-branes. As in the Klein Bottle case, we have the following cases:

  i. Only $R_3 \in G$:

  $$(1_{NS} - 1_R)\mathcal{V}_3 \frac{1}{\prod_l 2 \, \sin \pi v_\alpha^l} \Big(Tr[\gamma_{\alpha,9}] + \prod_l 2 \sin \pi v_\alpha^l \, Tr[\gamma_{\alpha,5_3}]\Big)^2 \, . \qquad (15)$$

  ii. Only $R_i \in G$ for a given $i = 1$ or 2:

  $$(1_{NS} - 1_R) \frac{1}{\prod_l 2 \, \sin \pi v_\alpha^l} \Big(\sqrt{\mathcal{V}_3} Tr[\gamma_{\alpha,9}] - \frac{1}{\sqrt{\mathcal{V}_3}} 2 \sin \pi v_\alpha^j \, Tr[\gamma_{\alpha,5_i}]\Big)^2 \, . \qquad (16)$$

  iii. All three Z_2 factors $R_l \in G$ with $l = 1, 2, 3$:

  $$(1_{NS} - 1_R) \frac{1}{\prod_l 2 \sin 2\pi v_\alpha^l} \Bigg[\sqrt{\mathcal{V}_3} \Big(Tr[\gamma_{\alpha,9}] + \prod_l 2 \sin \pi v_\alpha^l Tr[\gamma_{\alpha,5_3}]\Big)$$
  $$- \frac{1}{\sqrt{\mathcal{V}_3}} \sum_{i \neq j = 1,2} 2 \sin \pi v_\alpha^j Tr[\gamma_{\alpha,5_i}] \Bigg]^2 \, . \qquad (17)$$

The amplitudes are again proportional to zero: $(1_{NS} - 1_R)$, reflecting the fact that the orientifold group action preserve supersymmetry. The multiplicative factor is a function of the volume of the unaffected torus:

$$(1_{NS} - 1_R) \left[A_1\sqrt{\mathcal{V}_3} + \frac{A_2}{\sqrt{\mathcal{V}_3}}\right]^2 \, . \qquad (18)$$

A_1 and A_2 are functions of the traces of the Chan-Paton factors $Tr[\gamma_{\alpha,I}]$. A_1 is the contribution of the Newmann-Newmann strings longitudinal to the unaffected torus (they have NN boundary conditions in this torus), and is proportional to $Tr[\gamma_{\alpha,9}]$ and $Tr[\gamma_{\alpha,5_3}]$. Whereas, A_2 is the contribution of the Dirichlet-Dirichlet strings transverse to $\mathcal{V}_3$, and is function of $Tr[\gamma_{\alpha,5_i}]$ for $i = 1, 2$.

- For an element α such that $v_\alpha = (v_\alpha^1, v_\alpha^2, v_\alpha^3)$ the contribution to the tadpoles from the Annulus amplitudes are as follows:

  - when G contains no Z_2 factors, we have just the contribution of the 99 strings.

  $$(1_{NS} - 1_R) \frac{1}{\prod_l 2 \, \sin \pi v_\alpha^l} Tr[\gamma_{\alpha,9}]^2 \, . \qquad (19)$$

  - when G contains R factors, then:

    i. If there is only one $R_i \in G$ (for a given i), we have:

    $$(1_{NS} - 1_R) \frac{1}{\prod_l 2 \, \sin \pi v_\alpha^l} \Big(Tr[\gamma_{\alpha,9}] + \prod_{l \neq i} 2 \sin \pi v_\alpha^l \, Tr[\gamma_{\alpha,5_i}]\Big)^2 \, . \qquad (20)$$

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ii. If all three $R_l \in G$ (with $l = 1, 2, 3$), we should include its corresponding D5$_l$-branes:

$$(1_{NS} - 1_R)\frac{1}{\prod_l 2 \sin 2\pi v_\alpha^l}\left(Tr[\gamma_{\alpha,9}] + \sum_{i=1}^{3}\prod_{l \neq i} 2 \sin \pi v_\alpha^l Tr[\gamma_{\alpha,5_i}]\right)^2 . \quad (21)$$

The structure of these amplitudes is similar to (18) without the volume dependance and with the product extended over $l = 1, 2, 3$.

Möbius Strip

Finally, the contribution of the group element α to the Möbius strip amplitude can be computed from the trace formula:

$$\mathcal{M}_{I,\alpha} = Tr_I\left[\Omega\alpha\ q^{L_0}\right] , \quad (22)$$

where the trace is over open strings attached on the DI-brane. To extract the contribution to the tadpoles we must perform a modular transformation to the transverse channel by $P = TST^2ST$ where $T : \tau \to \tau + 1$ and $S : \tau \to -1/\tau$, $l = 1/8t$. Finally, we take the UV limit $l \to \infty$.

The Möbius strip transverse channel amplitude is also the mean value of the transverse channel Klein Bottle and Annulus amplitudes [2, 7]. Therefore, extracting the UV limit in the Möbius strip amplitude and comparing with the Klein Bottle and Annulus amplitudes, we obtain constraints on the matrices $\gamma_{\alpha,I}$ and $\gamma_{\Omega,\alpha,I}$:

$$Tr[\gamma_{\Omega\alpha,9}^T\gamma_{\Omega\alpha,9}^{-1}] = Tr[\gamma_{\alpha^2,9}] ,$$
$$Tr[\gamma_{\Omega R_i\alpha,9}^T\gamma_{\Omega R_i\alpha,9}^{-1}] = -Tr[\gamma_{\alpha^2,9}] ,$$
$$Tr[\gamma_{\Omega\alpha,5_i}^T\gamma_{\Omega\alpha,5_i}^{-1}] = -Tr[\gamma_{\alpha^2,5_i}] ,$$
$$Tr[\gamma_{\Omega R_i\alpha,5_i}^T\gamma_{\Omega R_i\alpha,5_i}^{-1}] = Tr[\gamma_{\alpha^2,5_i}] ,$$
$$Tr[\gamma_{\Omega R_j\alpha,5_i}^T\gamma_{\Omega R_j\alpha,5_i}^{-1}] = -Tr[\gamma_{\alpha^2,5_i}] , \quad (23)$$

where in the last equation $i \neq j$ and $i, j = 1, 2, 3$. These constraints appears to be the same for both $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$ and $v_\alpha = (v_\alpha^1, v_\alpha^2, v_\alpha^3)$. Details of the calculations are reported in the appendix.

Tadpole conditions:

The massless part of the transverse channel amplitudes $\tilde{\mathcal{K}}_\alpha + \tilde{\mathcal{A}}_\alpha + \tilde{\mathcal{M}}_\alpha$ gives the tadpole conditions. The tadpole conditions for a given element α^2 are:

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- If α is such that $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$, then:

  - when G contains no Z_2 factors:

$$Tr[\gamma_{\alpha^2,9}] = 32 \prod_l \cos \pi v_\alpha^l \ . \tag{24}$$

  - when G contains Z_2 factors, then we have the following cases:

    i. If only $R_3 \in G$:

$$Tr[\gamma_{\alpha^2,9}] + 4 \prod_l \sin 2\pi v_\alpha^l \ Tr[\gamma_{\alpha^2,5_3}] = 32 \left(\prod_l \cos \pi v_\alpha^l + \prod_l \sin \pi v_\alpha^l \right) . \tag{25}$$

    ii. If only $R_i \in G$ for a given $i = 1$ or 2:

$$\begin{aligned} Tr[\gamma_{\alpha^2,9}] &=& 32 \prod_l \cos \pi v_\alpha^l \ , \\ 2 \sin 2\pi v_\alpha^j \ Tr[\gamma_{\alpha^2,5_i}] &=& 32 \ \cos \pi v_\alpha^i \sin \pi v_\alpha^j \ . \end{aligned} \tag{26}$$

    iii. If all three $R_l \in G$ with $l = 1, 2, 3$:

$$\begin{aligned} Tr[\gamma_{\alpha^2,9}] + 4 \prod_l \sin 2\pi v_\alpha^l Tr[\gamma_{\alpha^2,5_3}] &=& 32 \left(\prod_l \cos \pi v_\alpha^l + \prod_l \sin \pi v_\alpha^l \right) , \\ \sum_{i \neq j = 1,2} 2 \sin 2\pi v_\alpha^j Tr[\gamma_{\alpha^2,5_i}] &=& 32 \sum_{i \neq j = 1,2} \epsilon_{ij} \cos \pi v_\alpha^i \sin \pi v_\alpha^j \ . \end{aligned} \tag{27}$$

- If α is such that $v_\alpha = (v_\alpha^1, v_\alpha^2, v_\alpha^3)$ then:

  - when G does not contain any Z_2 factors:

$$Tr[\gamma_{\alpha^2,9}] = 32 \prod_l \cos \pi v_\alpha^l \ , \tag{28}$$

  - when G does contain Z_2 factors, then:

    i. $R_i \in G$ for a given i:

$$\begin{aligned} Tr[\gamma_{\alpha^2,9}] &+& 4 \prod_{l \neq i} \sin 2\pi v_\alpha^l \ Tr[\gamma_{\alpha^2,5_i}] \\ &=& 32 \left(\prod_l \cos \pi v_\alpha^l + \cos \pi v_\alpha^i \prod_{l \neq i} \sin \pi v_\alpha^l \right) . \end{aligned} \tag{29}$$

    ii. $R_l \in G$ with $l = 1, 2, 3$:

$$\begin{aligned} Tr[\gamma_{\alpha^2,9}] &+& 4 \sum_{i=1}^{3} \prod_{l \neq i} \sin 2\pi v_\alpha^l Tr[\gamma_{\alpha^2,5_i}] \\ &=& 32 \left(\prod_l \cos \pi v_\alpha^l + \sum_i \cos \pi v_\alpha^i \prod_{l \neq i} \sin \pi v_\alpha^l \right) . \end{aligned} \tag{30}$$

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Note that all the tadpole conditions hold for both NS and R sectors according to super-symmetry.

The tadpole condition for group elements that are not square of some other element of G (there is no element $\beta \in G$ such that $\alpha = \beta^2$), will receive contribution only from the Annulus amplitude. When this element is such that $v = (v_\alpha^1, v_\alpha^2, 0)$ or $v = g_3 + v_\alpha$, the tadpole conditions will be the same as before (24-27), with zero on the right hand side. For elements such that $v = g_i + v_\alpha$ it is not difficult to work out the tadpole conditions, leading to:

$$
\begin{aligned}
Tr[\gamma_{R_i\alpha,9}] &+ 4\sin\pi v_\alpha^i \cos\pi v_\alpha^j Tr[\gamma_{R_i\alpha,5_3}] \\
&+ 2\cos\pi v_\alpha^j Tr[\gamma_{R_i\alpha,5_i}] + 2\sin\pi v_\alpha^i Tr[\gamma_{R_i\alpha,5_j}] = 0 \ ,
\end{aligned}
\tag{31}
$$

where $i \neq j = 1,2$ and the different terms exist only if the corresponding R_i factor does. If $v_\alpha = (v_\alpha^1, v_\alpha^2, v_\alpha^3)$, the tadpole conditions are the same as (28-30) without the right hand side (i.e. the right hand side is zero).

In Section 5 we will give some applications of the formulae we have obtained in this section and compare with the supersymmetric orientifolds studied in the literature.

4   Breaking Supersymmetry with momentum shifts

In this section we include the Scherk-Schwarz (SS) deformation which is a translation in one direction in the torus T_3^2 by a momentum shift of order two accompanied by $(-1)^F$ where F is the spacetime fermion number. This deformation is compatible only with an orbifold action that commutes with it, therefore, we will restrict ourselves to elements of the form $v = (v_1, v_2, 0)$ or $v + g_i$ where g_i is the shift vector of a Z_2 element that leaves the coordinates of T_i^2 invariant and gives a minus sign to the other compact coordinates.

Klein Bottle

Since Ω exchange the left and the right moving modes, it acts on the momentum and windings as:

$$
\Omega \, |m,n\rangle \ \rightarrow \ |m,-n\rangle \ .
\tag{32}
$$

Therefore, the invariant states are those with vanishing winding modes i.e. $n = 0$. The shift acts on the zero modes as $(-1)^m \Lambda_{m,n}$ where $\Lambda_{m,n}$ is the Narain lattice sum. It is easy to see that the h twisted sector with the zero mode part being $\Lambda_{m,n+\frac{1}{2}}$ does not survive the Ω projection. However, if Ω is acting together with a Z_2 element, the combined action

gives:

$$\Omega R \, |m, n\rangle \rightarrow \, |-m, n\rangle \, . \tag{33}$$

The invariant states in this case are those with vanishing momentum i.e. $m = 0$. There-fore, the h twisted sector survives this projection if h and R acts in the same direction.

It is not difficult to see that the tadpoles in this model correspond to $O5$ orientifold planes sitting on the R-fixed points (as in the previous section) and $\bar{O}5$ orientifold planes sitting on the Rh-fixed points [8, 10]. To extract the massless tadpole contribution we need to perform a modular transformation $l = 1/4t$ and then take $l \rightarrow \infty$. Except from (5-8) we will also have contributions from the h twisted sector as well as $R_i h$, they both contribute $(1_{NS} + 1_R)$ to the tadpoles reflecting the breaking of supersymmetry.

- When the orbifold group G does not contain a Z_2 element, the contribution to the Klein Bottle will result only from the untwisted sector as for the case without SS. There is no contribution from states $\mathcal{K}_{h\alpha}$ (4) due to the shift (it gives rise only to massive states). Therefore, the contribution is the same as the one obtained in the supersymmetric case (5).

- when G contains an R factor, we have also contributions from the R twisted states:

  i. If only $R_3 \in G$, the contribution is exactly as before (6) (without SS), because the Scherk-Schwarz deformation is acting transverse to R_3. The $R_3 h$ twisted states do not contribute since they are massive.

  ii. If only $R_i \in G$ for a given $i = 1$ or 2:

$$\frac{1}{\prod_l 2 \, \sin 2\pi v_\alpha^l} \left[1_{NS} \left(\sqrt{\mathcal{V}_3} \prod_l 2 \cos \pi v_\alpha^l - \frac{2}{\sqrt{\mathcal{V}_3}} 2 \cos \pi v_\alpha^i 2 \sin \pi v_\alpha^j \right)^2 \right.$$
$$\left. - 1_R \mathcal{V}_3 \left(\prod_l 2 \, \cos \pi v_\alpha^l \right)^2 \right] , \tag{34}$$

  iii. If all three $R_l \in G$ with $l = 1, 2, 3$:

$$\frac{1}{\prod_l 2 \sin 2\pi v_\alpha^l} \left\{ 1_{NS} \left[\sqrt{\mathcal{V}_3} \left(\prod_l 2 \cos \pi v_\alpha^l + \prod_l 2 \sin \pi v_\alpha^l \right) \right. \right.$$
$$\left. \left. - \frac{2}{\sqrt{\mathcal{V}_3}} \sum_{i \neq j = 1,2} \epsilon_{ij} \, 2 \cos \pi v_\alpha^i 2 \sin \pi v_\alpha^j \right]^2 - 1_R \mathcal{V}_3 \left(\prod_l 2 \, \cos \pi v_\alpha^l + \prod_l 2 \, \sin \pi v_\alpha^l \right)^2 \right] \right\} . \tag{35}$$

All the amplitudes are perfect squares as they should be. Moreover, the cases (ii.) and (iii.) do not appear as $(1_{NS} - 1_R)$. This dissimilarity of the coefficients of the NS and

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R oscillators is exactly the effect of SS deformation and the breaking of supersymmetry. All the amplitudes have the general form

$$1_{NS} \left[K_{NS,1} \sqrt{\mathcal{V}_3} + \frac{K_{NS,2}}{\sqrt{\mathcal{V}_3}} \right]^2 - 1_R \left[K_{R,1} \sqrt{\mathcal{V}_3} + \frac{K_{R,2}}{\sqrt{\mathcal{V}_3}} \right]^2 \,, \tag{36}$$

where $K_{NS,2} \sim (1+1)f(v_\alpha)$, $K_{R,2} \sim (1-1)f(v_\alpha) = 0$ and $f(v_\alpha)$ a function of the shift vector v_α. The second term in both $K_{NS,2}$ and $K_{R,2}$ is due to h twisted states. This explains the appearance of the factor of 2 in the NS sector in (34) and (35) and the absence of the factor proportional to $\frac{1}{\sqrt{\mathcal{V}_3}}$ in the R sector.

Annulus

To cancel these tadpoles one needs to add D9, D5$_3$ and D5$_i$-branes as well as $\bar{\text{D}}5_i$-antibranes when $R_i \in G$ with $i = 1, 2$, and the Scherk-Schwarz element h acts on the T_3^2 torus. The anti $\bar{\text{D}}5_i$-branes sit on the $R_i h$ fixed points [8].
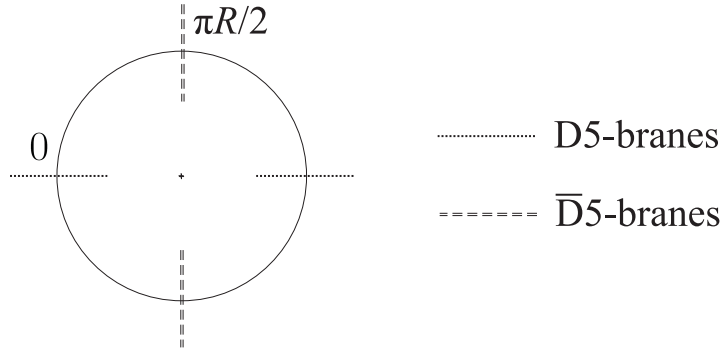
The contribution from the Annulus amplitudes are the same as in the supersymmetric case (without SS deformation) with in addition the anti $\bar{\text{D}}5_{i\neq3}$-brane sector when $R_i \in G$. Note that the Annulus amplitudes between two D-branes contributes $(1_{NS} - 1_R)$, whereas, the ones between a D-brane and an anti $\bar{\text{D}}$-brane leads $(1_{NS} + 1_R)$ reflecting the breaking of supersymmetry. It is not difficult to work out the contribution of an element α such that $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$:

- when G contains no Z_2 factors[5] the contribution from the annulus is the same as the one in the supersymmetric case (14).

- when G contains Z_2 factors, then:

    i. If only $R_3 \in G$, the contribution is the same as the one in the supersymmetric case (15).

---

[5]By Z_2 factors we refer to rotation elements and not to the Scherk-Schwarz deformation h.

ii. If only $R_i \in G$ for a given $i = 1$ or 2:

$$\frac{1}{\prod_l 2 \sin \pi v_\alpha^l} \times$$

$$\left[1_{NS}\left(\sqrt{\mathcal{V}_3} Tr[\gamma_{\alpha,9}] - \frac{1}{\sqrt{\mathcal{V}_3}} 2 \sin \pi v_\alpha^j \left(Tr[\gamma_{\alpha,5_i}] + Tr[\gamma_{\alpha,\bar{5}_i}] \right) \right)^2 \right.$$

$$\left. - 1_R\left(\sqrt{\mathcal{V}_3} Tr[\gamma_{\alpha,9}] - \frac{1}{\sqrt{\mathcal{V}_3}} 2 \sin \pi v_\alpha^j \left(Tr[\gamma_{\alpha,5_i}] - Tr[\gamma_{\alpha,\bar{5}_i}] \right) \right)^2 \right] . \quad (37)$$

iii. If all possible $R_l \in G$, with $l = 1, 2, 3$:

$$\frac{1}{\prod_l 2 \sin 2\pi v_\alpha^l} \left[1_{NS}\left(\sqrt{\mathcal{V}_3}\left(Tr[\gamma_{\alpha,9}] + \prod_l 2 \sin \pi v_\alpha^l Tr[\gamma_{\alpha,5_3}] \right) \right. \right.$$

$$\left. - \frac{1}{\sqrt{\mathcal{V}_3}} \sum_{i \neq j = 1,2} 2 \sin \pi v_\alpha^j \left(Tr[\gamma_{\alpha,5_i}] + Tr[\gamma_{\alpha,\bar{5}_i}] \right) \right)^2$$

$$- 1_R\left(\sqrt{\mathcal{V}_3}\left(Tr[\gamma_{\alpha,9}] + \prod_l 2 \sin \pi v_\alpha^l Tr[\gamma_{\alpha,5_3}] \right) \right.$$

$$\left. \left. - \frac{1}{\sqrt{\mathcal{V}_3}} \sum_{i \neq j = 1,2} 2 \sin \pi v_\alpha^j \left(Tr[\gamma_{\alpha,5_i}] - Tr[\gamma_{\alpha,\bar{5}_i}] \right) \right)^2 \right] . (38)$$

The general form, is a function of the volume of the unaffected by α torus ($\mathcal{V}_3$):

$$1_{NS}\left[A_{NS,1}\sqrt{\mathcal{V}_3} + \frac{A_{NS,2}}{\sqrt{\mathcal{V}_3}} \right]^2 - 1_R\left[A_{R,1}\sqrt{\mathcal{V}_3} + \frac{A_{R,2}}{\sqrt{\mathcal{V}_3}} \right]^2 , \quad (39)$$

where A_1 and A_2 are again functions of the traces of the Chan-Paton factors $Tr[\gamma_{\alpha,I}]$. $A_{NS,1}$ and $A_{R,1}$ are proportional to $Tr[\gamma_{\alpha,9}]$ and $Tr[\gamma_{\alpha,5_3}]$. $A_{NS,2}$ and $A_{R,2}$ proportional to $Tr[\gamma_{\alpha,5_i}]$ and $Tr[\gamma_{\alpha,\bar{5}_i}]$ for $i = 1, 2$.

Möbius Strip

Finally, the Möbius strip amplitude derived in two inequivalent ways (as a direct amplitude and as the mean value of the Klein Bottle and the Annulus amplitudes) leads the same constraints as before (23) with in addition:

- When $R_{i \neq 3} \notin G$, then:

i. If $R_3 \notin G$:

$$Tr[\gamma_{\Omega h\alpha,9}^T \gamma_{\Omega h\alpha,9}^{-1}] = \pm Tr[\gamma_{\alpha^2,9}] , \quad (40)$$

ii. If $R_3 \in G$ we have in addition:

$$Tr[\gamma_{\Omega h\alpha,5_3}^T \gamma_{\Omega h\alpha,5_3}^{-1}] = \pm Tr[\gamma_{\alpha^2,5_3}] . \quad (41)$$

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We should take the same sign for the D9 and D5$_3$ sectors by T-duality. Examples of this cases have been discussed in [11].

- When $R_{i\neq 3} \in G$ for a given $i = 1$ or 2:

$$Tr[\gamma^T_{\Omega h\alpha,I}\gamma^{-1}_{\Omega h\alpha,I}] = Tr[\gamma_{\alpha^2,I}], \qquad I = 9, 5_3, 5_i, \bar{5}_i \ . \tag{42}$$

In all cases $\gamma^2_{R,I} = -1$ with $I = 9, 5_l, \bar{5}_i$ for all R's.

The details of the calculations are reported in the appendices.

Tadpole conditions:

The tadpole conditions for an element α^2 such that $v_\alpha = (v^1_\alpha, v^2_\alpha, 0)$ are as follows:

- When G contains no Z_2 factors, the tadpole condition is the same as in the case without SS deformation (24):

$$Tr[\gamma_{\alpha^2,9}] = 32 \prod_l \cos \pi v^l_\alpha \ . \tag{43}$$

- When G contains Z_2 factors:

  i. If only $R_3 \in G$, the tadpole condition is the same as in the case without SS deformation (25):

$$Tr[\gamma_{\alpha^2,9}] + 4 \prod_l \sin 2\pi v^l_\alpha \ Tr[\gamma_{\alpha^2,5_3}]$$
$$= 32 \left(\prod_l \cos \pi v^l_\alpha + \prod_l \sin \pi v^l_\alpha \right) \ . \tag{44}$$

  ii. If only $R_i \in G$ for a given $i = 1$ or 2:

$$Tr[\gamma_{\alpha^2,9}] = 32 \prod_l \cos \pi v^l_\alpha \ ,$$

$$1_{NS}: \quad 2 \sin 2\pi v^j_\alpha \left(Tr[\gamma_{\alpha^2,5_i}] + Tr[\gamma_{\alpha^2,\bar{5}_i}] \right) = 32 \ \cos \pi v^i_\alpha \ \sin \pi v^j_\alpha \ ,$$

$$1_R: \quad 2 \sin 2\pi v^j_\alpha \left(Tr[\gamma_{\alpha^2,5_i}] - Tr[\gamma_{\alpha^2,\bar{5}_i}] \right) = 0 \ . \tag{45}$$

  iii. If all possible $R_l \in G$ with $l = 1, 2, 3$:

$$Tr[\gamma_{\alpha^2,9}] + 4 \prod_l \sin 2\pi v^l_\alpha Tr[\gamma_{\alpha^2,5_3}] = 32 \left(\prod_l \cos \pi v^l_\alpha + \prod_l \sin \pi v^l_\alpha \right) \ ,$$

$$1_{NS}: \quad 2 \sum_{i\neq j=1,2} \sin 2\pi v^j_\alpha \left(Tr[\gamma_{\alpha^2,5_i}] + Tr[\gamma_{\alpha^2,\bar{5}_i}] \right)$$

$$= 32 \sum_{i\neq j=1,2} \epsilon_{ij} \; \cos \pi v_\alpha^i \sin \pi v_\alpha^j \; ,$$

$$1_R: \qquad 2 \sum_{i\neq j=1,2} \sin 2\pi v_\alpha^j \Big(Tr[\gamma_{\alpha^2,5_i}] - Tr[\gamma_{\alpha^2,\bar{5}_i}] \Big) = 0 \; . \tag{46}$$

Finally, there could be elements that can not be expressed as the square of any other element in G. These elements will not receive contribution from the Klein Bottle amplitude. For such elements and for elements of the form $R_3\alpha$ the tadpole conditions are the same as (43-46) with zero on the right hand side. For the group elements $h\alpha$ and $R_3h\alpha$ the tadpole conditions are as (43) and (44) and since D5$_i$ and $\bar{\text{D}}5_i$ are transverse to the direction where h acts, there are no conditions on $Tr[\gamma_{h\alpha,5_i}]$ and $Tr[\gamma_{h\alpha,\bar{5}_i}]$. For the group element $R_i\alpha$ the tadpole condition is

$$
\begin{aligned}
Tr[\gamma_{R_i\alpha,9}] \quad &+ \quad 4\sin\pi v_\alpha^i \cos\pi v_\alpha^j Tr[\gamma_{R_i\alpha,5_3}] \\
&+ \quad 2\cos\pi v_\alpha^j Tr[\gamma_{R_i\alpha,5_i}] + 2\sin\pi v_\alpha^i Tr[\gamma_{R_i\alpha,5_j}] = 0 \; ,
\end{aligned}
\tag{47}
$$

where the $\bar{\text{D}}5_i$-branes do not contribute because they do not sit on the fixed points of this element. This condition is valid for both NS and R sectors (multiplied by $(1_{NS} - 1_R)$). For the element $R_ih\alpha$ we find

$$
\begin{aligned}
1_{NS}: \quad Tr[\gamma_{R_ih\alpha,9}] \quad &+ \quad 4\sin\pi v_\alpha^i \cos\pi v_\alpha^j Tr[\gamma_{R_ih\alpha,5_3}] \\
&+ \quad 2\cos\pi v_\alpha^j Tr[\gamma_{R_i\alpha,\bar{5}_i}] + 2\sin\pi v_\alpha^i Tr[\gamma_{R_ih\alpha,\bar{5}_j}] = 0 \; , \\
1_R: \quad Tr[\gamma_{R_ih\alpha,9}] \quad &+ \quad 4\sin\pi v_\alpha^i \cos\pi v_\alpha^j Tr[\gamma_{R_ih\alpha,5_3}] \\
&- \quad 2\cos\pi v_\alpha^j Tr[\gamma_{R_ih\alpha,\bar{5}_i}] - 2\sin\pi v_\alpha^i Tr[\gamma_{R_ih\alpha,\bar{5}_j}] = 0 \; ,
\end{aligned}
\tag{48}
$$

where the D5$_i$-branes do not contribute because they do not sit on the fixed points of $R_ih\alpha$.

5 Some specific examples

Let us discuss some examples that can be described by the general formulae we have obtained in the previous sections.

The first example is the orbifold groups studied by Gimon and Johnson [6], where $G = Z_N$ for $N = 2, 3, 4, 6$ acting on T^4. The tadpole conditions are given by (43-44) with $v_\alpha^1 = -v_\alpha^2 = \frac{k}{N}$, $v_\alpha^3 = 0$ leading for odd N

$$Tr[\gamma_{2k,9}] = 32\cos^2\frac{k}{N}\pi \; , \tag{49}$$

whereas for even N:

$$Tr[\gamma_{2k,9}] - 4\;\sin^2\frac{2k}{N}\pi\; Tr[\gamma_{2k,5_3}] = 32\;\cos\frac{2k}{N}\pi \; ,$$

$$Tr[\gamma_{2k-1,9}] - 4 \ \sin^2 \frac{2k-1}{N}\pi \ Tr[\gamma_{2k-1,5_3}] = 0 \ . \tag{50}$$

The tadpole conditions for the groups studied by Zwart [12], where $G = Z_N \times Z_M$, can be easily reproduced by using (24-31). As an example the tadpole conditions for $Z_N \times Z_M$ where both N and M odd are given by:

$$\begin{aligned}
Tr[\gamma_{2k,0,9}] &= 32 \ \cos^2 \frac{k}{N}\pi \ , \\
Tr[\gamma_{0,2l,9}] &= 32 \ \cos^2 \frac{l}{M}\pi \ , \\
Tr[\gamma_{2k,2l,9}] &= 32 \ \cos \frac{k}{N}\pi \ \cos \frac{l}{M}\pi \ \cos \left(\frac{l}{M} - \frac{k}{N}\right)\pi \ ,
\end{aligned} \tag{51}$$

all the other cases can be easily worked out in a similar way. The tadpole conditions for the groups studied by Ibanez et al [13], where $G = Z_N$ with both N even and odd and $G = Z_N \times Z_M$ with N or M even, can be reproduced using (24-31) by specifying the vector v_α for the different group elements.

In [12, 13] it was found that the tadpole conditions for Z_4, Z_8, Z_8' and Z_{12}' have no solutions. In particular, for Z_4 with $v_\alpha = (\frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$ the tadpole conditions can be written as:

$$\begin{aligned}
Tr[\gamma_{\alpha,9}] + 2Tr[\gamma_{\alpha,5_3}] &= 0 \ , \\
Tr[\gamma_{\alpha^2,9}] + 4Tr[\gamma_{\alpha^2,5_3}] &= 0 \ , \\
Tr[\gamma_{\alpha^3,9}] + 2Tr[\gamma_{\alpha^3,5_3}] &= 0 \ ,
\end{aligned} \tag{52}$$

plus a contribution in the Klein Bottle of the form (93) proportional to $1/\mathcal{V}_3$ that can not be cancelled by any D-brane contribution. This is a general feature for all the inconsistent groups. They contain elements α of the form $\alpha = \beta R_i$ where $v_\beta = (v_\beta^1, v_\beta^2, 0)$ and $R_i \notin G$ contributing a $\frac{1}{\mathcal{V}_3}$ to the tadpoles. Therefore, the introduction of D9 and D5$_3$-branes contributing $\mathcal{V}_3$ alone is not enough to cancel all the tadpoles. In [14] it has been argued that these orientifolds contain non-perturbative sectors that are missing in the worldsheet approach. Attempts to cancel these tadpoles has been given in several papers [1, 2, 7, 15, 16] by adding discrete torsion or Wilson lines. In the next section we will solve the tadpole conditions for the non-supersymmetric version of Z_4, where supersymmetry is broken by an order two Scherk-Schwarz deformation and show that the obtained spectrum is chiral and anomaly free.

6   Solving the tadpole conditions

Open string states are denoted by $|\psi, ab\rangle$, where ψ refers to the world-sheet degrees of freedom while a, b are Chan-Paton indices associated to open string endpoints lying on

D-branes. The action of a group element α on this state is given by

$$\alpha : |\psi, ab\rangle \quad \rightarrow \quad (\gamma_\alpha)_{a,a'} |\alpha\psi, a'b'\rangle (\gamma_\alpha)^{-1}_{b'b} , \tag{53}$$

where γ_α is a unitary matrix associated to α. the action of $\Omega\alpha$ is given by

$$\Omega\alpha : |\psi, ab\rangle \quad \rightarrow \quad (\gamma_{\Omega\alpha})_{a,a'} |\alpha\psi, b'a'\rangle (\gamma_{\Omega\alpha})^{-1}_{b'b} , \tag{54}$$

The Chan-Paton matrices must be such that the full states are invariant under the action of the orientifold group. Hence,

$$\lambda^{(i)} \sim e^{2\pi i b^i_\alpha} \; \gamma_a \lambda^{(i)} \gamma^{-1}_\alpha , \qquad \lambda^{(i)} \sim \gamma_\Omega \lambda^{(i)T} \gamma^{-1}_\Omega . \tag{55}$$

A simple way to impose the tadpole conditions on the Chan-Paton matrices λ is to recast them in a Cartan-Weyl basis. In this case, constraints on λ's will emerge as restrictions on weight vectors [13]. They can be organized into charged generators: $\lambda_a = E_a$ and Cartan generators $\lambda_I = H_I$ such that:

$$[H_I, E_a] = \rho^a_I E_a , \tag{56}$$

where $\rho^a_I = (\underline{\pm 1, \pm 1, 0, \cdots, 0})$ are the roots associated to the generators E_a of the $SO(32)$ Lie-algebra. The underline denotes all the possible permutations. The matrix γ_α and its powers represent the action of the orientifold group on the Chan-Paton factors, and they correspond to elements of a discrete subgroup of the Abelian group spanned by the Cartan generators. Hence, we can write:

$$\gamma_\alpha = e^{-2\pi i V_\alpha \cdot H} , \tag{57}$$

where the dimension of the *shift* vector V_α is the rank of the $SO(32)$ Lie-group. Cartan generators are represented by 2×2 σ_3 submatrices.

Recalling the formula $e^{-B} A e^B = \sum^\infty_{n=0} [A, B]_n$ with $[A, B]_{n+1} = [[A, B]_n, B]$ and $[A, B]_0 = A$, and using (56), it is easy to show that:

$$\gamma_\alpha E_a \gamma^{-1}_\alpha = e^{-2\pi i \rho_a \cdot V_\alpha} E_a , \tag{58}$$

The equations giving the massless spectrum (55) can be expressed in the following way:

$$\rho^a_I \cdot V_\alpha = b_\alpha , \tag{59}$$

where "b_α" is associated with the transformation of the corresponding massless fields. Notice the difference between 99, $5_i 5_i$ and 95_i strings:

$$b_{99,5_i5_i} = \begin{cases} 0 & \text{for vectors} \\ v^i_\alpha & \text{for scalars} \\ s \cdot v_\alpha & \text{for fermions} \end{cases} , \quad b_{95_i} = \begin{cases} s_j v_j + s_l v_l & \text{for scalars} \\ s_i v_i & \text{for fermions} \end{cases} \tag{60}$$

where b_α's should be understood modulo $\mathbb{Z}$.

In general, the matrix $\gamma_{\alpha,p}$ which acts on a given Dp-brane, satisfy:

$$\gamma_\alpha^k = \pm 1 \ , \tag{61}$$

where k is the order of the associated orientifold group element α, and it is the smallest integer such that $\alpha^k = 1$. Depending on the sign choice in (61), the vector V_α has the form

$$V_\alpha = \frac{1}{k}\left(0^{n_0}, 1^{n_1}, \cdots, j^{n_j}, \cdots\right) \ , \tag{62}$$

for the minus sign, where $j = 0, \cdots, k-1$ and

$$V_\alpha = \frac{1}{2k}\left(1^{n_1}, 3^{n_2}, \cdots, (2j-1)^{n_j}, \cdots\right) \ , \tag{63}$$

for the plus sign, where $j = 0, \cdots, \frac{k}{2}$. The number of each entry is determined by the corresponding tadpole condition.

The (pq) sector is handled using an auxiliary $SO(64) \supset SO(32)_{(pp)} \otimes SO(32)_{(qq)}$ algebra. Since we have generators acting simultaneously on both Dp and Dq branes, only roots of the form:

$$\rho_{(pq)} = \rho_{(p)} \otimes \rho_{(q)} = (\underline{\pm 1, 0, \ldots, 0}; \underline{\pm 1, 0, \ldots, 0}) \ , \tag{64}$$

must be considered. The shift vector is than defined as

$$W_{(pq)} = V_{(p)} \otimes V_{(q)} \ . \tag{65}$$

In [17] the full anomaly free open string spectrum for odd and even order (with one Z_2 factor) orientifolds was found for a twist vector given by $\frac{1}{N}(t_1, t_2, t_3)$ where $\sum_i t_i = 0$. We would like to study orientifold models with more than one Z_2 namely two Z_2 factors and show that we can use the same techniques for most cases, namely $Z_2 \times Z_2 \times G$ where G is a group with no Z_2 elements and work out the open string spectrum.

Supersymmetric $Z_2 \times Z_2 \times Z_N$

The supersymmetric $Z_2 \times Z_2$ orientifold model was solved in [18] where the gauge group was found to be $USp(16)$ with scalars in the antisymmetric representation $\square = 120$.

Following [17] we will be more general and consider also Z_N orbifold actions that are not necessarily crystallographic. A closed inspection of the tadpole conditions shows that for N odd the Z_N matrices γ_α commutes with all the Z_2 matrices γ_{R_i}, therefore, to solve the tadpole conditions we can use the Cartan-Weyl basis. For the spectrum we have:

- Gauge bosons:

$$\rho_I^a \cdot V_\alpha = 0 \ , \tag{66}$$

where now ρ_I^a are the eight dimensional roots associated with the generators of $USp(16)$ Lie algebra. These are $\rho_I^a = (\pm 1, \pm 1, 0, \cdots, 0)$ together with the long roots $\rho_I^a = (\pm 2, 0, \cdots, 0)$.

- Scalars:

$$\rho_I^a \cdot V_\alpha = b_\alpha \ , \tag{67}$$

where b_α is given in (60) and ρ_I^a are the roots associated with the generators of the antisymmetric representation of $USp(16)$, namely, $\rho_I^a = (\pm 1, \pm 1, 0, \cdots, 0)$.

General Massless formulae

Using the technic described above, we can work out the general massless formulae for the case $Z_2 \times Z_2 \times Z_N$ with N odd and shift vector acting as $(v_\alpha^1, v_\alpha^2, 0)$ [6]:

- For 99/55 states, we have:

  ▷ Vectors in: $USp(n_1) \times \prod_{i=2}^{(N+1)/2} U(n_i)$.

  ▷ Scalars $\psi_{-1/2}^I |0\rangle$ in: $(n_i, n_{2-i+b_\alpha^I N})$, $(n_i, \overline{n}_{i-b_\alpha^I N})$, $(\overline{n}_i, \overline{n}_{2-i-b_\alpha^I N})$.
    Representations of $USp(n_1)$ appear as $n_1 + \bar{n}_1$ that denote the vector $n_{1,v}$. There will be antisymmetric reps in the $U(n_i)$ iff $2i - 2 = v_I N$.

- For the $95_i/5_j 5_k$ states we have:

  ▷ Scalars $|s_1, s_2\rangle$ $(95_3/5_1 5_2)$ in: $(n_1, \tilde{n}_1)$, $(n_i, \overline{\tilde{n}}_i)$, $(\overline{n}_i, \tilde{n}_i)$,

  ▷ Scalars $|s_1, s_3\rangle$ $(95_2/5_1 5_3)$ in: $(n_i, \tilde{n}_{2-i+b_\alpha^2 N})$, $(n_{2-i+b_\alpha^2 N}, \tilde{n}_i)$, $(n_i, \overline{\tilde{n}}_{i-b_\alpha^2 N})$, $(\overline{n}_{i-b_\alpha^2 N}, \tilde{n}_i)$, $(\overline{n}_i, \overline{\tilde{n}}_{2-i-b_\alpha^2 N})$, $(\overline{n}_{2-i-b_\alpha^2 N}, \overline{\tilde{n}}_i)$.

  ▷ Scalars $|s_2, s_3\rangle$ $(95_1/5_2 5_3)$ in: $(n_i, \tilde{n}_{2-i+b_\alpha^1 N})$, $(n_{2-i+b_\alpha^1 N}, \tilde{n}_i)$, $(n_i, \overline{\tilde{n}}_{i-b_\alpha^1 N})$, $(\overline{n}_{i-b_\alpha^1 N}, \tilde{n}_i)$, $(\overline{n}_i, \overline{\tilde{n}}_{2-i-b_\alpha^1 N})$, $(\overline{n}_{2-i-b_\alpha^1 N}, \overline{\tilde{n}}_i)$.

Fermions will transform in similar representations due to supersymmetry (up to now we discuss only the action of rotation elements onto the Chan-Paton factors).

---

[6]The n_i's are defined by the tadpole conditions.

Scherk-Schwarz deformation

Scherk-Schwarz deformation acts on the open strings in the same way as the rotation elements do. The shift vector corresponding to $\gamma_h^2 = \pm 1$ is given by:

$$V_h = \frac{1}{4} \begin{cases} (1_a, -1_b) & \text{for } \gamma_h^2 = -1 \\ (2_a, 0_b) & \text{for } \gamma_h^2 = +1 \end{cases} \tag{68}$$

where the index refer to the number of components in the vector. The action of the Scherk-Schwarz deformation is as (55) with:

$$b_h = \begin{cases} 0 & \text{for spacetime bosons} \\ 1/2 & \text{for spacetime fermions} \end{cases} \tag{69}$$

where b_h is defined modulo $\mathbb{Z}$.

When the SS deformation acts on a supersymmetric model, it breaks the gauge group for $\gamma_h^2 = -1$, as:

$$U(N) \rightarrow U(n) \times U(N-n) \,, \tag{70}$$

and for both $G = SO(2N), USp(2N)$

$$G \rightarrow U(N) \,, \tag{71}$$

whereas for $\gamma_h^2 = +1$ as:

$$G_N \rightarrow G_n \times G_{N-n} \,, \tag{72}$$

where $G_n = U(n), SO(n)$ and $USp(n)$. The remaining representations split accordingly for $\gamma_h^2 = -1$:

$$(m, n) \rightarrow \begin{cases} (m_1, n_2) + (m_2, n_1) & \text{bosons} \\ (m_1, n_1) + (m_2, n_2) & \text{fermions} \end{cases} \tag{73}$$

whereas for $\gamma_h^2 = +1$:

$$(m, n) \rightarrow \begin{cases} (m_1, n_1) + (m_2, n_2) & \text{bosons} \\ (m_1, n_2) + (m_2, n_1) & \text{fermions} \end{cases} \tag{74}$$

and for both $\gamma_h^2 = \pm 1$ the bifundamental representations split as:

$$(m, \bar{n}) \rightarrow \begin{cases} (m_1, \bar{n}_1) + (m_2, \bar{n}_2) & \text{bosons} \\ (m_1, \bar{n}_2) + (m_2, \bar{n}_1) & \text{fermions} \end{cases} \tag{75}$$

where $m = m_1 + m_2$ and $n = n_1 + n_2$.

Non-Supersymmetric Z_4

Consider the non-supersymmetric version of Z_4 orientifold by adding a SS deformation. The orbifold group is generated by a twist whose action on the three complex coordinates is given by $v_\alpha = (\frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$. The undesired aforementioned Klein-bottle R-R tadpoles cancel between (93) and (95). This is true also for Z_8, Z_8' and Z_{12}. The remaining tadpole conditions are the same as before (52) with the same conditions for $h\alpha$ where $\gamma_{h,9}^2 = \gamma_{h,5_3}^2 = +1$. Their cancellation requires $Tr\gamma_\alpha = Tr\gamma_{h\alpha} = 0$ for all elements α of Z_4. Hence, They can be easily solved with the matrices:

$$\begin{aligned}
\gamma_\alpha &= \text{diag}[\phi\mathbf{I}_a, \phi^{-1}\mathbf{I}_a, \phi\mathbf{I}_b, \phi^{-1}\mathbf{I}_b, \phi^3\mathbf{I}_c, \phi^{-3}\mathbf{I}_c, \phi^3\mathbf{I}_d, \phi^{-3}\mathbf{I}_d] \ , \\
\gamma_h &= \text{diag}[-\mathbf{I}_a, -\mathbf{I}_a, \mathbf{I}_b, \mathbf{I}_b, -\mathbf{I}_c, -\mathbf{I}_c, \mathbf{I}_d, \mathbf{I}_d] \ ,
\end{aligned} \tag{76}$$

where $a + b = c + d = 8$, $\phi = e^{2\pi i/8}$ and $\gamma_\alpha^4 = -\mathbf{I}$. The open string massless spectrum is given in Table 1, it is chiral and does not suffer of irreducible gauge anomalies.

Another non-supersymmetric version of this model can be obtained by using the projection $\Omega' = \Omega \cdot J$ (instead of Ω), which gives a relative sign between the untwisted and twisted states in the Klein bottle [7]. This action is called "discrete torsion" and it introduces O^+ orientifold planes instead of O^-. To cancel the resulting R-R tadpoles the introduction of D9 and anti $\bar{\text{D}}5$-branes (instead of D5-branes) are needed. In terms of Chan-Paton matrices this action is reflected as a constraint on the γ's by requiring them to satisfy $\gamma_\alpha^4 = +\mathbf{I}$. The matrices which satisfy all the tadpole conditions are:

$$\begin{aligned}
\gamma_\alpha &= \text{diag}[i\mathbf{I}_a, -i\mathbf{I}_a, i\mathbf{I}_b, -i\mathbf{I}_b, -\mathbf{I}_c, -\mathbf{I}_d, \mathbf{I}_e, \mathbf{I}_f] \\
\gamma_h &= \text{diag}[-\mathbf{I}_a, -\mathbf{I}_a, \mathbf{I}_b, \mathbf{I}_b, -\mathbf{I}_c, \mathbf{I}_d, -\mathbf{I}_e, \mathbf{I}_f]
\end{aligned} \tag{77}$$

where $c + d = e + f$. The spectrum is again non-chiral and anomaly free (Table 1).

Non-Supersymmetric $Z_6 \times Z_2$

Another example that we consider is the non-supersymmetric version of the $Z_6 \times Z_2$ orientifold by SS deformation. The supersymmetric version was discussed by Zwart [12]. The shift vectors of the different elements are:

$$v_\alpha = \frac{1}{3}(-1, 1, 0) \ , \quad g_1 = \frac{1}{2}(0, -1, 1) \ , \quad g_2 = \frac{1}{2}(1, 0, -1) \ . \tag{78}$$

where there is another Z_2 element with shift vector $g_3 = g_1 + g_2$. The Ω action on the D9-branes is described by a symmetric matrix that we choose to be:

$$\gamma_{\Omega,9} = \mathbf{I}_{32} \ . \tag{79}$$

Z₄× SS $(\gamma_\alpha^4 = -\mathbf{I})$		
$U(a) \times U(b) \times$ $U(c) \times U(d)_{9,5}$	(99)/(55) matter	(95) matter
Bosons	$2\left((\bar{a},c) + (\bar{b},d) + \boxbar_a + \boxbar_b + \bar{\boxbar}_c + \bar{\boxbar}_d\right) +$ $(\bar{a},\bar{c}) + (a,c) + (\bar{b},\bar{d}) + (b,d)$	$(a;a) + (\bar{a};c) + (b;b) + (\bar{b};d)+$ $(\bar{c};\bar{c}) + (c;\bar{a}) + (d;\bar{b}) + (\bar{d};\bar{d})$
Fermions	$(a,\bar{b}) + (\bar{c},d) + c.c.+$ $2\left((a,b) + (\bar{a},d) + (\bar{b},c) + (\bar{c},\bar{d})\right)$ $(a,d) + (\bar{a},\bar{d}) + (b,c) + (\bar{b},\bar{c})$	$(a;b) + (\bar{a};d) + (b;a) + (\bar{b};c)+$ $(c;\bar{b}) + (\bar{c};\bar{d}) + (d;\bar{a}) + (\bar{d};\bar{c})$
Z₄× SS $(\gamma_\alpha^4 = +\mathbf{I})$		
$U(a) \times U(b) \times SO(c)$ $SO(d) \times SO(e) \times SO(f)_{9,5}$	(99)/(55) matter	(95) matter
Bosons	$2\left((\bar{a},c) + (a,e) + (\bar{b},d) + (b,f)\right) +$ $\boxbar_a + \bar{\boxbar}_a + \boxbar_b + \bar{\boxbar}_b + (c,e) + (d,f)$	$(\bar{a};d) + (a;f) + (\bar{b};c) + (b;e)+$ $(c;\bar{b}) + (d;\bar{a}) + (e;b) + (f;a)$
Fermions	$(a,\bar{b}) + (\bar{a},b) + (c,d) + (e,f)+$ $2\left((\bar{a},d) + (a,f) + (\bar{b},c) + (b,e)\right)$ $(a,b) + (\bar{a},\bar{b}) + (c,f) + (d,e)$	$(\bar{a};d) + (a;f) + (\bar{b};c) + (b;e)+$ $(c;\bar{b}) + (d;\bar{a}) + (e;b) + (f;a)$

Table 1: The massless spectrum of the non supersymmetric Z_4 models.

Supersymmetry is broken by introducing a SS deformation h acting on T_3^2 with a shift of order two. By studying the Klein bottle amplitudes we realize that there are two O^--planes sitting on the $R_3 = R_1 \cdot R_2$ fixed points. The R_1 and R_2 elements act in the same direction as the SS deformation h giving O^--planes sitting on the two R-fixed points and $\bar{O}^-$-planes on the Rh-fixed points. To cancel the tadpoles we need to add $\mathrm{D}5_i$-branes on the R_i-fixed points with $i = 1,2,3$ and $\bar{\mathrm{D}}5_i$ antibranes on the R_ih-fixed points with $i = 1,2$. The γ_{R_l} matrices satisfying the tadpole conditions, are:

$$\gamma_{R_1,9} = \begin{pmatrix} 0 & -\mathbf{I} \\ \mathbf{I} & 0 \end{pmatrix} , \quad \gamma_{R_2,9} = i\begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix} , \quad \gamma_{R_3,9} = -i\begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix} \tag{80}$$

where $\epsilon = \mathbf{I}_2 \otimes \sigma_2 \otimes \mathbf{I}_4$. The Z_3 action on the Chan Paton matrices is given by the reducible matrix:

$$\gamma_{\alpha,9} = \mathrm{diag}[A_4, A_4, \mathbf{I}_8, A_4, A_4, \mathbf{I}_8] \tag{81}$$

where $A_4 = \mathbf{I}_2 \otimes \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}$ and SS action on the Chan-Paton is given by:

$$\gamma_{h,9} = \mathrm{diag}[\mathbf{I}_4, -\mathbf{I}_8, \mathbf{I}_8, -\mathbf{I}_8, \mathbf{I}_4] \tag{82}$$

where $\gamma_{h,9}^2 = +\mathbf{I}$. The gauge group for the D9 branes is $U(2) \times U(2) \times USp(4) \times USp(4)$.

For the D5 branes, γ_Ω is antisymmetric, therefore, we can choose:

$$\gamma_{\Omega,5_i} = i\sigma_2 \otimes \mathbf{I}_{16} \ . \tag{83}$$

for all $i = 1, 2, 3$. The orbifold action is then:

$$\gamma_{R_i,5_i} = (-1)^{\delta_{3,i}} \begin{pmatrix} 0 & -\mathbf{I} \\ \mathbf{I} & 0 \end{pmatrix} \ , \quad \gamma_{R_i,5_{i+1}} = \begin{pmatrix} \mathbf{I} & 0 \\ 0 & -\mathbf{I} \end{pmatrix} \ , \quad \gamma_{R_i,5_{i+2}} = \begin{pmatrix} 0 & -\mathbf{I} \\ -\mathbf{I} & 0 \end{pmatrix} \tag{84}$$

where the indexes are defined modulo 3. The Z_3 action on the Chan Paton is given by:

$$\gamma_{\theta,5_i} = (-1)^{\delta_{2,i}} \mathrm{diag}[\phi\mathbf{I}_2, \phi^{-1}\mathbf{I}_2, \phi\mathbf{I}_2, \phi^{-1}\mathbf{I}_2, \mathbf{I}_8, \phi\mathbf{I}_2, \phi^{-1}\mathbf{I}_2, \phi\mathbf{I}_2, \phi^{-1}\mathbf{I}_2, \mathbf{I}_8] \ . \tag{85}$$

where $\phi = e^{2\pi i/3}$. The SS action on the D5$_i$ Chan Paton matrices are the same as the D9 one $\gamma_{h,5_i} = \gamma_{h,9}$.

The spectrum is chiral and anomaly free (Table 2). Few comments are in order: a) Since h acts longitudinal to D9 and D5$_3$ branes, their corresponding open strings are affected by the action of the SS deformation. Hence, the spectrum appears to be non-supersymmetric. b) Moreover, since h acts transverse to D5$_1$, D5$_2$, $\bar{\text{D}}5_1$ and $\bar{\text{D}}5_2$ their corresponding open strings are not affected by the SS deformation. The spectrum in these cases is supersymmetric. c) Notice the T-dualities exchanging D9 $\leftrightarrow$ D5$_3$, D5$_1$ $\leftrightarrow$ D5$_2$ and $\bar{\text{D}}5_1 \leftrightarrow \bar{\text{D}}5_2$ branes. The open string spectrum that we found does indeed obey these duality symmetries.

In this chapter we analytically solve the tadpole conditions and we evaluate the massless spectrum of the Z_4 and the $Z_6 \times Z_2$ orientifolds with a Scherk-Schwarz deformation. However, another way to obtain the same solutions is by using the formulae given in chapter 6.

7   Conclusion

In this paper we have computed the tadpole conditions for the orientifold projection $G + \Omega G$ of type IIB string theory on T^6 for both supersymmetric and non-supersymmetric cases, where G contains a Scherk-Schwarz deformation that shifts the momenta and gives different boundary conditions to bosons and fermions. We have found that, when an element that breaks supersymmetry acts in the same direction as a Z_2 reflection element, anti-D5-branes must be added to cancel the tadpoles.

It is easy to generalize our results to cases for which the breaking of supersymmetry is made by a winding shift (instead of momentum shift). In that case, the adding of anti-$\bar{\text{D}}9$-branes is necessary to cancel the tadpoles. Anti-$\bar{\text{D}}5$-branes should be included when a

$\mathbf{Z}_6 \times \mathbf{Z}_2$+SS		
Sectors	**Bosons**	**Fermions**
99/ $5_3 5_3$	$U(2) \times U(2) \times USp(4) \times USp(4)$ $\bar{\Box}_a + \bar{\Box}_b + (2,1,1,4) + (1,2,4,1)$ $\Box_a + \Box_b + (\bar{2},1,1,4) + (1,\bar{2},4,1)$ +Scalars in the adjoints	$2\,((\bar{2},2,1,1) + (2,\bar{2},1,1) + (1,1,4,4))$ $(2,2,1,1) + (\bar{2},1,4,1) + (1,\bar{2},1,4)$ $(\bar{2},\bar{2},1,1) + (2,1,4,1) + (1,2,1,4)$
$5_1 5_1/ \bar{5}_1\bar{5}_1/$ $5_2 5_2/ \bar{5}_2\bar{5}_2$ (SUSY)	$U(4) \times USp(8)$ $(\Box,1) + (\bar{\Box},1) + (\bar{4},8) + (4,8)$ Scalars in the Adjoint	$(\Box,1) + (\bar{\Box},1) + (\bar{4},8) + (4,8)$ 2 Fermions in the Adjoint
$95_1/ 5_3 5_2$	$(\bar{2},1,1,1;\bar{4},1) + (2,1,1,1;1,8) +$ $(1,1,1,4;4,1)$	$(1,2,1,1;4,1) + (1,\bar{2},1,1;1,8) +$ $(1,1,4,1;\bar{4},1)$
$9\bar{5}_1/ 5_3 \bar{5}_2$	$(\bar{2},1,1,1;\bar{4},1) + (2,1,1,1;1,8) +$ $(1,1,1,4;4,1)$	$(1,2,1,1;4,1) + (1,\bar{2},1,1;1,8) +$ $(1,1,4,1;\bar{4},1)$
$95_2/ 5_3 5_1$	$(2,1,1,1;4,1) + (\bar{2},1,1,1;1,8)$ $+(1,1,1,4;\bar{4},1)$	$(1,\bar{2},1,1;\bar{4},1) + (1,2,1,1;1,8)$ $+(1,1,4,1;4,1)$
$9\bar{5}_2/ 5_3 \bar{5}_1$	$(2,1,1,1;4,1) + (\bar{2},1,1,1;1,8)$ $+(1,1,1,4;\bar{4},1)$	$(1,\bar{2},1,1;\bar{4},1) + (1,2,1,1;1,8)$ $+(1,1,4,1;4,1)$
95_3	$(2,1,1,1;\bar{2},1,1,1) + (\bar{2},1,1,1;2,1,1,1)$ $(1,2,1,1;1,\bar{2},1,1) + (1,\bar{2},1,1;1,2,1,1)$ $(1,1,4,1;1,1,4,1) + (1,1,1,4;1,1,1,4)$	$(2,1,1,1;1,\bar{2},1,1) + (\bar{2},1,1,1;1,2,1,1)$ $(1,2,1,1;\bar{2},1,1,1) + (1,\bar{2},1,1;2,1,1,1)$ $(1,1,4,1;1,1,1,4) + (1,1,1,4;1,1,4,1)$
$5_1 5_2/ \bar{5}_1\bar{5}_2$ (SUSY)	$(4,1;\bar{4},1) + (\bar{4},1;4,1)$ $(1,8;1,8)$	$(4,1;\bar{4},1) + (\bar{4},1;4,1)$ $(1,8;1,8)$

Table 2: The massless spectrum of the $Z_6 \times Z_2$ with a Scherk-Schwarz deformation. The spectrum is chiral and anomaly free. Notice the supersymmetric and not supersymmetric sectors.

Z_2 reflection element acts perpendicular to the element that breaks supersymmetry. The tadpole conditions for the antibranes are similar to the ones for the branes.

We have found complete agreement with models studied in the literature and by adding an order two Scherk-Schwarz deformation to the supersymmetric T^6/Z_4 model we was able to solve the tadpole conditions and obtain an anomaly free massless spectrum. We argued that the same calculations can be done for the other inconsistent groups Z_8, Z_8' and Z_{12}.

In all the paper we have only considered D-branes sitting on one fixed point, mainly the origin. However, it is not difficult to study more general cases where the D5-branes are distributed on different fixed points. We did not consider the additional freedom of adding Wilson lines, they are non dynamical. A non-trivial effective potential is generated for these gauge invariant operators that dynamically breaks part of the gauge group.

This study and classification of supersymmetric and non-supersymmetric orientifolds can be very useful for model building [19]. This analysis can be extended to more general orientifold groups $G_1 + \Omega G_2$, as it was discussed in the introduction, by considering group elements which do not commute with Ω as well as asymmetric orbifold groups [20]. We can also study models with fluxes which are T-dual to orientifolds with branes at angles. The last appear to be particularly interesting for phenomenological applications [21, 22, 23, 24, 25, 26, 27, 28].

Acknowledgments

A   Some definitions

Let us define some of the objects that we used in this paper. The oscillator dependant parts are:

$$T\begin{bmatrix}0\\v\end{bmatrix} = \frac{1}{2\eta^2}\sum_{a,b}(-1)^{a+b+ab}\frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}}{\eta}\prod_i -2\sin\pi v_i\frac{\vartheta\begin{bmatrix}a\\b+2v_i\end{bmatrix}}{\vartheta\begin{bmatrix}1\\1+2v_i\end{bmatrix}}. \tag{86}$$

$$
T[^u_v] \;=\; \frac{1}{2\eta^2} \sum_{a,b} (-1)^{a+b+ab} \frac{\vartheta[^a_b]}{\eta} \prod_i \frac{\vartheta[^{a+2u_i}_{b+2v_i}]}{\vartheta[^{1+2u_i}_{1+2v_i}]}. \tag{87}
$$

The lattice parts are:

$$
\Lambda_{m+a,n+b} = \frac{1}{\eta(q)\eta(\bar q)} \sum_{m,n} q^{\frac{\alpha'}{4}\left(\frac{m+a}{R}+\frac{n+b}{\alpha'}R\right)^2} \bar q^{\frac{\alpha'}{4}\left(\frac{m+a}{R}+\frac{n+b}{\alpha'}R\right)^2} \tag{88}
$$

and the momentum and winding parts:

$$
P_m(i\tau_2/2) \;=\; \frac{1}{\eta(i\tau_2/2)} \sum_m q^{\frac{\alpha'}{4}\left(\frac{m}{R}\right)^2} \tag{89}
$$

$$
W_n(i\tau_2/2) \;=\; \frac{1}{\eta(i\tau_2/2)} \sum_n q^{\frac{\alpha'}{4}\left(\frac{nR}{\alpha'}\right)^2} \tag{90}
$$

B   Orientifolds

In the appendix we will give the complete structure of the formulae we have used in the main paper.

B.1   Klein Bottle

Consider an element $\alpha \in G$ such that $v_\alpha = (v^1_\alpha, v^2_\alpha, 0)$. The contribution to the Klein Bottle amplitude of this element $\alpha \in G$ can be written in the form:

$$
\mathcal{K}_\alpha \sim T[^0_{2v_\alpha}] + T[^g_{2v_\alpha}]. \tag{91}
$$

In all expressions we skip the integral $\frac{1}{2}\int \frac{d\tau_2}{\tau_2^3}$. To extract the massless tadpole contribution we need to perform a modular transformation $l = 1/4t$ where t is the loop modulus and l the cylinder length and then take the limit $l \to \infty$. After taking this limit we find:

$$
T[^0_{2v_\alpha}] \to (1_{NS} - 1_R)\mathcal{V}_3 \frac{2^3}{\prod_l 2 \; \sin 2\pi v^l_\alpha} \left(\prod_l 2 \; \cos \pi v^l_\alpha\right)^2, \tag{92}
$$

where the factor 2^3 comes from the modular transformation. If the orbifold group G contains a Z_2 factor (denoted by R), it will give an extra contribution since $(\Omega R\alpha)^2 = \alpha^2$:

$$
T[^0_{2(g_i v_\alpha)}] \to -(1_{NS} - 1_R)\frac{1}{\mathcal{V}_3} \frac{2^3}{\prod_l 2 \; \sin 2\pi v^l_\alpha} (2 \; \cos \pi v^i_\alpha)^2 (2 \; \sin \pi v^j_\alpha)^2 \;,\;\; i \neq j = 1,2, \tag{93}
$$

$$
T[^0_{2(g_3 v_\alpha)}] \to (1_{NS} - 1_R)\mathcal{V}_3 \frac{2^3}{\prod_l 2 \; \sin 2\pi v^l_\alpha} \left(\prod_l 2 \; \sin \pi v^l_\alpha\right)^2. \tag{94}
$$

When we also include an order two Scherk-Schwarz deformation h, we will have contributions from the twisted sector by h whenever there is a Z_2 element acting in the same direction. The contribution of these sectors to the tadpoles is given by

$$T[_{2(g_i v_\alpha)}^{\quad h}] = T[_{2(g_i h v_\alpha)}^{\quad h}] \rightarrow -(1_{NS} + 1_R)\frac{2^3}{\mathcal{V}_3}\frac{1}{\prod_l 2\ \sin 2\pi v_\alpha^l}(2\ \cos \pi v_\alpha^i)^2(2\ \sin \pi v_\alpha^j)^2, \quad (95)$$

where as before $i \neq j = 1, 2$. There are also twisted sectors by $R_i h$ that will produce similar results. Taking this into account we can easily work out the equations provided in section 4.

Putting all these together we find the amplitudes in the different cases presented in the body of the text (5-12).

B.2   Annulus

To cancel the tadpoles we need to include D-branes in the spectrum (open string sector). They are a bunch of D9-branes and in case the group G contains g_i-factors we also need D5$_i$-branes extended along the T_i^2 torus and siting on the R_i-fixed points. The contribution of an element α can be written in the form $\frac{1}{2}\int \frac{d\tau_2}{\tau_2^3}$:

$$\mathcal{A}_\alpha = \left(Tr[\gamma_{\alpha,9}]^2 + Tr[\gamma_{\alpha,5_i}]^2\right)T[_{v_\alpha}^0] + 2Tr[\gamma_{v_\alpha,9}]Tr[\gamma_{a,5_i}]T[_{v_\alpha}^{g_i}] \quad (96)$$

To extract the tadpole contributions we need to perform a modular transformation to the transverse channel $l = 1/2t$ and then take the limit $l \rightarrow \infty$ [5]. If $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$ then, in the UV limit we find:

$$\mathcal{A}_{99,\alpha} \rightarrow (1_{NS} - 1_R)\mathcal{V}_3\frac{2^{-3}}{\prod_l 2\ \sin \pi v_\alpha^l}Tr[\gamma_{\alpha,9}]^2. \quad (97)$$

If the group G contains an R-factor than we will also have to consider the corresponding D5-brane sectors fixed under α:

$$\mathcal{A}_{5_i5_i,\alpha} \rightarrow (1_{NS} - 1_R)\frac{2^{-3}}{\mathcal{V}_3}\frac{1}{\prod_l 2\ \sin \pi v_\alpha^l}(2\sin \pi v_\alpha^j)^2\ Tr[\gamma_{\alpha,5_i}]^2. \quad (98)$$

where $i \neq j = 1, 2$, and

$$\mathcal{A}_{5_35_3,\alpha} \rightarrow (1_{NS} - 1_R)\mathcal{V}_3\frac{2^{-3}}{\prod_l 2\ \sin \pi v_\alpha^l}\left(\prod_l 2\sin \pi v_\alpha^l\right)^2 Tr[\gamma_{\alpha,5_3}]^2. \quad (99)$$

Including the open strings ending on different types of D-branes we find the amplitudes (14-21).

To cancel the Klein Bottle tadpoles corresponding to the Scherk-Schwarz deformation h we need to add $\bar{D}5_i$ on $R_i h$ fixed points when $R_i \in G$. The Annulus amplitudes between

two D-branes contributes $(1_{NS} - 1_R)$ whereas, the ones between a D-brane and an anti D-brane leads $(1_{NS} + 1_R)$ reflecting the breaking of supersymmetry. The contribution to the tadpoles from the $\bar{D}5_i$ branes is given by

$$\mathcal{A}_{\bar{5}_i\bar{5}_i,\alpha} \to (1_{NS} - 1_R)\frac{1}{\mathcal{V}_3}\frac{2^{-3}}{\prod_l 2 \sin \pi v_\alpha^l}(2 \sin \pi v_\alpha^j)^2 \; Tr[\gamma_{\alpha,\bar{5}_i}]^2. \tag{100}$$

B.3 Möbius Strip

Finally, the contribution to the Möbius strip amplitude of an element α of G can be written in the form:

$$\begin{aligned}
\mathcal{M}_\alpha \;=\; & -\Big(Tr[\gamma_{\Omega\alpha,9}^T\gamma_{\Omega\alpha,9}^{-1}]T[^0_{v_\alpha}] + Tr[\gamma_{\Omega R_i\alpha,9}^T\gamma_{\Omega R_i\alpha,9}^{-1}]T[^0_{g_iv_\alpha}] \\
& + Tr[\gamma_{\Omega\alpha,5_i}^T\gamma_{\Omega\alpha,5_i}^{-1}]T[^0_{g_iv_\alpha}] + Tr[\gamma_{\Omega g_iv,5_i}^T\gamma_{\Omega g_iv,5_i}^{-1}]T[^0_{v_\alpha}]\Big) \;.
\end{aligned}$$

where we skip again $\frac{1}{2}\int\frac{d\tau_2}{\tau_2^3}$. The overall minus sign is conventional. We must make the same choice of sign as for the identity element of G. To extract the tadpole conditions we must perform a modular transformation to the transverse channel by $P = TST^2ST$ where $T : \tau \to \tau + 1$ and $S : \tau \to -1/\tau$ going to the $l = 1/8t$. Finally, we take the UV limit $l \to \infty$. It is not difficult to work out the massless contributions in the different cases.

- For an element $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$:

  - In case G does not contain a Z_2 factor, the massless contribution is:

  $$-2(1_{NS} - 1_R)\; \mathcal{V}_3\;\frac{1}{\prod_l 2sin2\pi v_\alpha^l}Tr[\gamma_{\Omega\alpha,9}^T\gamma_{\Omega\alpha,9}^{-1}]\prod_l 2\cos \pi v_\alpha^l.$$

  - When G contains an R element, we should consider also the contribution coming from the corresponding D5-branes:

    i. If G contains only a R_3 element:

    $$\begin{aligned}
    -2\mathcal{V}_3\frac{(1_{NS} - 1_R)}{\prod_l 2\sin 2\pi v_\alpha^l}\Big\{& Tr[\gamma_{\Omega\alpha,9}^T\gamma_{\Omega\alpha,9}^{-1}]\prod_l 2\cos \pi v_\alpha^l - Tr[\gamma_{\Omega R_3\alpha,9}^T\gamma_{\Omega R_3\alpha,9}^{-1}]\prod_l 2\sin \pi v_\alpha^l \\
    & -\Big(Tr[\gamma_{\Omega\alpha,5_3}^T\gamma_{\Omega\alpha,5_3}^{-1}]\prod_l 2\cos \pi v_\alpha^l - Tr[\gamma_{\Omega R_3\alpha,5_3}^T\gamma_{\Omega R_3\alpha,5_3}^{-1}]\prod_l 2\sin \pi v_\alpha^l\Big)\prod_n 2\sin 2\pi v_\alpha^n\Big\}
    \end{aligned}$$

    ii. If only $R_i \in G$ for a given $i = 1$ or 2:

    $$\begin{aligned}
    -2\frac{(1_{NS} - 1_R)}{\prod_l 2sin2\pi v_\alpha^l}\Big\{& \mathcal{V}_3 Tr[\gamma_{\Omega\alpha,9}^T\gamma_{\Omega\alpha,9}^{-1}]\prod_l 2\cos \pi v_\alpha^l \\
    & + Tr[\gamma_{\Omega R_i\alpha,9}^T\gamma_{\Omega R_i\alpha,9}^{-1}]2\cos \pi v_\alpha^i 2\sin \pi v_\alpha^j + \Big(Tr[\gamma_{\Omega\alpha,5_i}^T\gamma_{\Omega\alpha,5_i}^{-1}]\prod_l 2\cos \pi v_\alpha^l \\
    & + \frac{1}{\mathcal{V}_3}Tr[\gamma_{\Omega R_i\alpha,5_i}^T\gamma_{\Omega R_i\alpha,5_i}^{-1}]2\cos \pi v_\alpha^i 2\sin \pi v_\alpha^j\Big)2\sin 2\pi v_\alpha^j\Big\}
    \end{aligned}$$

iii. If all three possible $R_l \in G$ with $l = 1, 2, 3$:

$$-2\frac{(1_{NS} - 1_R)}{\prod_l 2 sin 2\pi v_\alpha^l} \times$$

$$\left\{ \mathcal{V}_3 \left[Tr[\gamma_{\Omega\alpha,9}^T \gamma_{\Omega\alpha,9}^{-1}] \prod_l 2 \cos \pi v_\alpha^l - Tr[\gamma_{\Omega R3\alpha,9}^T \gamma_{\Omega R3\alpha,9}^{-1}] \prod_l 2 \sin \pi v_\alpha^l \right. \right.$$

$$\left. - \left(Tr[\gamma_{\Omega\alpha,5_3}^T \gamma_{\Omega\alpha,5_3}^{-1}] \prod_l 2 \cos \pi v_\alpha^l - Tr[\gamma_{\Omega R3\alpha,5_3}^T \gamma_{\Omega R3\alpha,5_3}^{-1}] \prod_l 2 \sin \pi v_\alpha^l \right) \prod_n 2 \sin 2\pi v_\alpha^n \right]$$

$$+ \frac{1}{\mathcal{V}_3} \sum_{i \neq j=1,2} \left(Tr[\gamma_{\Omega R_i\alpha,5_i}^T \gamma_{\Omega R_i\alpha,5_i}^{-1}] 2 \cos \pi v_\alpha^i 2 \sin \pi v_\alpha^j \right.$$

$$\left. - Tr[\gamma_{\Omega R_j\alpha,5_i}^T \gamma_{\Omega R_j\alpha,5_i}^{-1}] 2 \sin \pi v_\alpha^i 2 \cos \pi v_\alpha^j \right) 2 \sin 2\pi v_\alpha^j$$

$$+ \sum_{i \neq j=1,2} Tr[\gamma_{\Omega R_i\alpha,9}^T \gamma_{\Omega R_i\alpha,9}^{-1}] 2 \cos \pi v_\alpha^i 2 \sin \pi v_\alpha^j$$

$$+ \sum_{i \neq j=1,2} Tr[\gamma_{\Omega R_i\alpha,5_3}^T \gamma_{\Omega R_i\alpha,5_3}^{-1}] 2 \cos \pi v_\alpha^i 2 \sin \pi v_\alpha^j \prod_l 2 \sin 2\pi v_\alpha^l$$

$$+ \sum_{i \neq j=1,2} Tr[\gamma_{\Omega\alpha,5_i}^T \gamma_{\Omega\alpha,5_i}^{-1}] \prod_l 2 \cos \pi v_\alpha^l 2 \sin 2\pi v_\alpha^j$$

$$+ \left. \sum_{i \neq j=1,2} Tr[\gamma_{\Omega R3\alpha,5_i}^T \gamma_{\Omega R3\alpha,5_i}^{-1}] 2 \sin \pi v_\alpha^i 2 \sin \pi v_\alpha^j 2 \sin 2\pi v_\alpha^j \right\}.$$

- For an element $v_\alpha = (v_\alpha^1, v_\alpha^2, v_\alpha^3)$:

  - When G contains no R factors we have:

  $$-2(1_{NS} - 1_R) \frac{1}{\prod_l 2 \sin 2\pi v_\alpha^l} Tr[\gamma_{\Omega\alpha,9}^T \gamma_{\Omega\alpha,9}^{-1}] \prod_l 2 \cos \pi v_\alpha^l.$$

  - If G contains R factors:

    i. If there is only one $R_i \in G$ for a given i:

    $$-2 \frac{(1_{NS} - 1_R)}{\prod_l 2 \sin 2\pi v_\alpha^l} \left\{ Tr[\gamma_{\Omega\alpha,9}^T \gamma_{\Omega\alpha,9}^{-1}] \prod_l 2 \cos \pi v_l \right.$$

    $$- Tr[\gamma_{\Omega R_i\alpha,9}^T \gamma_{\Omega R_i\alpha,9}^{-1}] 2 \cos \pi v_\alpha^i \prod_{l \neq i} 2 \sin \pi v_\alpha^l - \left(Tr[\gamma_{\Omega\alpha,5_i}^T \gamma_{\Omega\alpha,5_i}^{-1}] \prod_l 2 \cos \pi v_\alpha^l \right.$$

    $$\left. \left. - Tr[\gamma_{\Omega R_i\alpha,5_i}^T \gamma_{\Omega R_i\alpha,5_i}^{-1}] 2 \cos \pi v_\alpha^i \prod_{l \neq i} 2 \sin \pi v_\alpha^l \right) \prod_{k \neq i} 2 \sin 2\pi v_\alpha^k \right\}$$

    ii. If all possible $R_l \in G$ with $l = 1, 2, 3$:

    $$-2 \frac{(1_{NS} - 1_R)}{\prod_l 2 \sin 2\pi v_\alpha^l} \times$$

$$\left\{ Tr[\gamma_{\Omega\alpha,9}^T \gamma_{\Omega\alpha,9}^{-1}] \prod_l 2\cos\pi v_\alpha^l - \sum_{i=1}^3 Tr[\gamma_{\Omega R_i\alpha,9}^T \gamma_{\Omega R_i\alpha,9}^{-1}] 2\cos\pi v_\alpha^i \prod_{l\neq i} 2\sin\pi v_\alpha^l \right.$$

$$- \sum_{i=1}^3 \left(Tr[\gamma_{\Omega\alpha,5_i}^T \gamma_{\Omega\alpha,5_i}^{-1}] \prod_l 2\cos\pi v_\alpha^l \right.$$

$$\left. - Tr[\gamma_{\Omega R_i\alpha,5_i}^T \gamma_{\Omega R_i\alpha,5_i}^{-1}] 2\cos\pi v_\alpha^i \prod_{l\neq i} 2\sin\pi v_\alpha^l \right) \left. \prod_{k\neq i} 2\sin 2\pi v_\alpha^k \right\}$$

Requiring that the Möbius strip transverse channel amplitude to be the mean value between the transverse channel Klein Bottle and Annulus amplitudes gives us the constraints (23) on the matrices $\gamma_{\alpha,I}$ and $\gamma_{\Omega.\alpha,I}$.

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